Chapter 9 section 2

Learing objective: to use rigid motions to transform objects in the coordinate plane.

Reflection: a mirror image of an object. The object is "reflected" across an axis.

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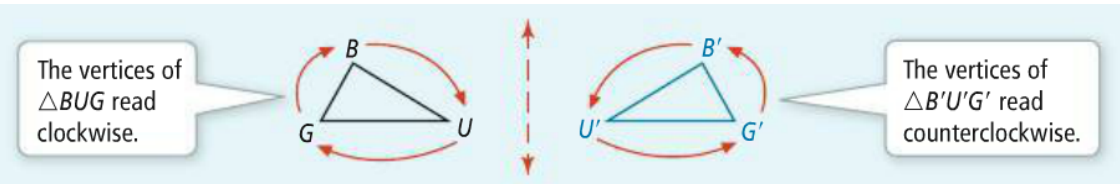

When you reflect an object across an axis the order of the vertices changes. Note how each vertex is exactly the same distance from the axis on both sides!

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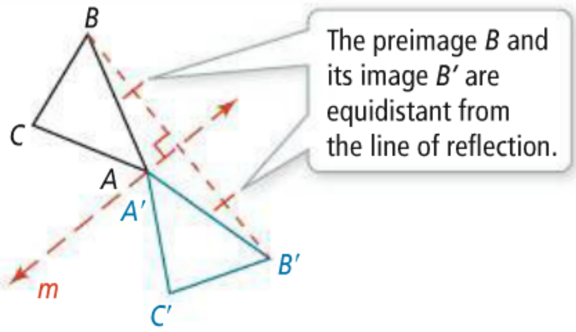
A **reflection** across a line m, called the **line of reflection,** is a transformation with the following properties:

- If a point A is on line m, then the image of A is itself (that is, $A' = A$).
- If a point B is not on line m, then m is the perpendicular bisector of $\overline{BB'}$.

You write the reflecion across m that takes P to P' as $R_m(P) = P'$.



R = reflection
m = axis of reflection
P = preimage
P' = image

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Multiple Choice Point P has coordinates $(3, 4)$. What are the coordinates of $R_{y=1}(P)$?

A $(3, -4)$ B $(0, 4)$ C $(3, -2)$ D $(-3, -2)$

Solution:
Step 1: graph the problem!
Step 2: either visually find the distance to the axis of reflection or use the distance formula. Then plot each vertex the same distance across the axis of reflection.

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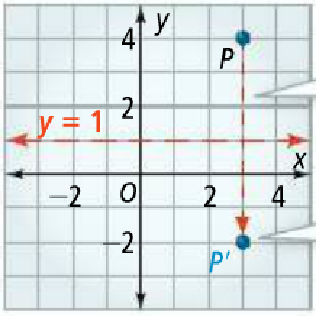
Multiple Choice Point P has coordinates (3, 4). What are the coordinates of $R_{y=1}(P)$?

(A) (3, −4) (B) (0, 4) (C) (3, −2) (D) (−3, −2)

Graph point P and the line of reflection $y = 1$. P and its reflection image across the line must be equidistant from the line of reflection.



Move along the line through P that is perpendicular to the line of reflection.

Stop when the distances of P and P' to the line of reflection are the same.

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Multiple Choice Point P has coordinates (3, 4). What are the coordinates of $R_{y=1}(P)$?

(A) (3, −4) (B) (0, 4) (C) (3, −2) (D) (−3, −2)

Algebraic solution:

Step 1: find the distance from the vertex to the axis of reflection.

Step 2: compute the new coordinate using this distance.

Original coordinate is (3,4). The axis of reflection is y=1. The y-coordinate 4 is 3 away from 1. So the new y-coordinate must also be 3 away from 1 in the other direction. 1 - 3 is -2, so the new y-coordinate must be -2.

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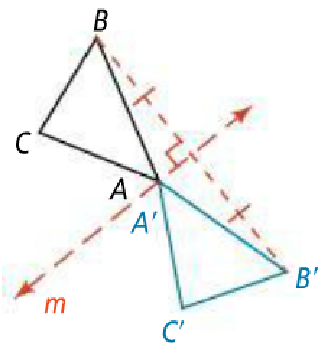
Property **Properties of Reflections**

- Reflections preserve distance.
  If $R_m(A) = A'$, and $R_m(B) = B'$, then $AB = A'B'$.
- Reflections preserve angle measure.
  If $R_m(\angle ABC) = \angle A'B'C'$, then $m\angle ABC = m\angle A'B'C'$.
- Reflections map each point of the preimage to one and only one corresponding point of its image.
  $R_m(A) = A'$ if and only if $R_m(A') = A$.



These properties mean that reflection is a *rigid motion*.

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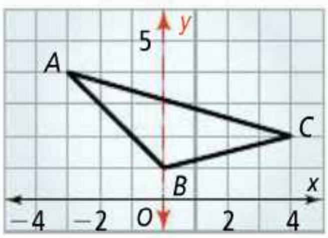
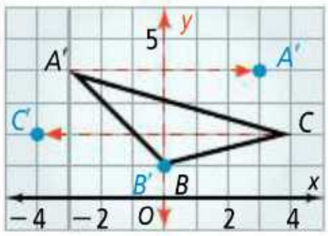
Coordinate Geometry **Graph points $A(-3, 4)$, $B(0, 1)$, and $C(4, 2)$. Graph and label $R_{y\text{-axis}}(\triangle ABC)$.**

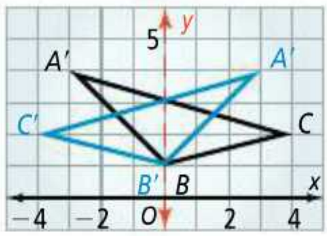
Step 1
Graph $\triangle ABC$. Show the y-axis as the dashed line of reflection.

Step 2
Find A', B', and C'. B' is in the same position as B because B is on the line of reflection. Locate A' and C' so that the y-axis is the perpendicular bisector of $\overline{AA'}$ and $\overline{CC'}$.

Step 3
Draw $\triangle A'B'C'$.







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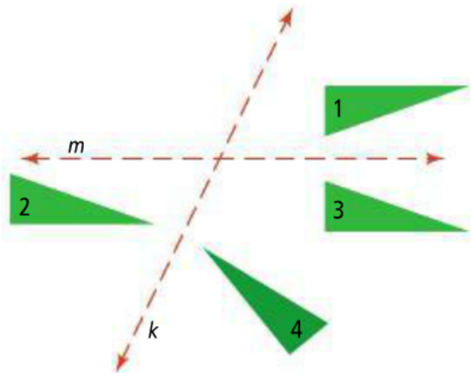
Writing reflection rules

Each triangle in the diagram is a reflection of another triangle across one of the given lines. How can you describe Triangle 2 by using a reflection rule?

Triangle 2 is the image of a reflection, so find the preimage and the line of reflection to write a rule.

The preimage cannot be Triangle 3 because Triangle 2 and Triangle 3 have the same orientation and reflections reverse orientation.



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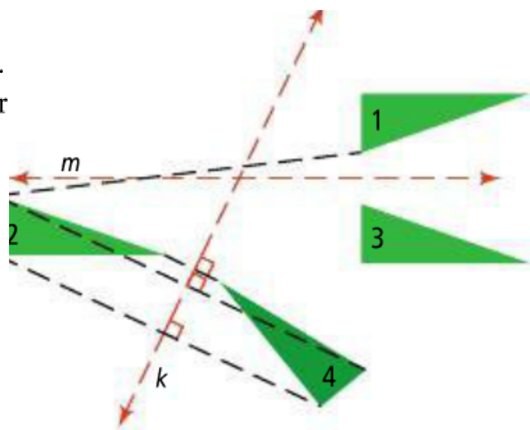
Writing reflection rules

Check Triangles 1 and 4 by drawing line segments that connect the corresponding vertices of Triangle 2. Because neither line k nor line m is the perpendicular bisector of the segment drawn from Triangle 1 to Triangle 2, Triangle 1 is not the preimage.

Line k is the perpendicular bisector of the segments joining corresponding vertices of Triangle 2 and Triangle 4. So, Triangle 2 = R_k(Triangle 4).



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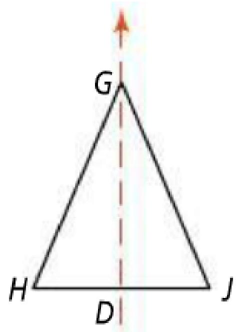
Example

In the diagram, $R_t(G) = G$, $R_t(H) = J$, and $R_t(D) = D$. Use the properties of reflections to describe how you know that $\triangle GHJ$ is an isosceles triangle.

Since $R_t(G) = G$, $R_t(H) = J$, and reflections preserve distance, $R_t(\overline{GH}) = \overline{GJ}$. So, $GH = GJ$ and, by definition, $\triangle GHJ$ is an isosceles triangle.



Key point: distance to the axis of reflection is the essential element of reflection!

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